Chapter 5. Rational Expressions

5.1. Simplify Rational Expressions

**KYOTE Standards:** CR 13; CA 7

**Definition 1.** A rational expression is the quotient \( \frac{P}{Q} \) of two polynomials \( P \) and \( Q \) in one or more variables, where \( Q \neq 0 \).

Some examples of rational expressions are:

\[
\begin{align*}
\frac{x}{x^2 + 2x - 4} & \quad \frac{ab^2}{c^4} & \quad \frac{x^3 - y^3}{x^2 + y^2}
\end{align*}
\]

Rational expressions in algebra are closely related to rational numbers in arithmetic as their names and definitions suggest. Recall that a rational number is the quotient \( \frac{p}{q} \) of two integers \( p \) and \( q \), where \( q \neq 0 \), as stated in Section 1.3. The procedure used to simplify a rational expression by dividing out the greatest common factor of the numerator and denominator is the same as the procedure used to reduce a rational number (a fraction) to lowest terms. The procedures used to multiply, divide, add and subtract rational expressions are the same as the corresponding procedures used to multiply, divide, add and subtract rational numbers.

**Example 1.** Find the greatest common factor (GCF) of the numerator and the denominator of each rational expression. Write the expression by factoring out the GCF in both the numerator and the denominator. Then divide out the GCF to write the rational expression in simplified form.

\[\begin{align*}
(a) \quad & \frac{18a^2b^5c^3}{12a^4b} \\
(b) \quad & \frac{9x^2(2x-1)^3}{24x^5(2x-1)}
\end{align*}\]

**Solution.** (a) The GCF of the numerator and denominator of \( \frac{18a^2b^5c^3}{12a^4b} \) is \( 6a^2b \). We factor out the GCF of the numerator and denominator and divide it out to obtain

\[
\frac{18a^2b^5c^3}{12a^4b} = \frac{6a^2b \cdot 3b^4c^3}{6a^2b \cdot 2a^2} = \frac{3b^4c^3}{2a^2}
\]

Divide out (cancel) \( 6a^2b \).
(b) The GCF of the numerator and denominator of \( \frac{9x^2(2x-1)^3}{24x^5(2x-1)} \) is \( 3x^2(2x-1) \).

We factor out the GCF of the numerator and denominator and divide it out to obtain

\[
\frac{9x^2(2x-1)^3}{24x^5(2x-1)} = \frac{3x^2(2x-1) \cdot 3(2x-1)^2}{3x^2(2x-1) \cdot 8x^3} = \frac{3(2x-1)^2}{8x^3}
\]

Factor \( 3x^2(2x-1) \) from numerator and denominator

Divide out (cancel) \( 3x^2(2x-1) \)

Example 2. Simplify the given rational expression.

(a) \( \frac{3a^2b}{3a^2b + 6ab} \)

(b) \( \frac{x^2 - x - 6}{3x^2 + 5x - 2} \)

(c) \( \frac{a^2b^3 - a^2b}{a^2b + a^2} \)

(d) \( \frac{y^2 - x^2}{x^2 - xy} \)

Solution. (a) We factor the denominator of \( \frac{3a^2b}{3a^2b + 6ab} \) and divide out the GCF \( 3ab \) of numerator and denominator to obtain

\[
\frac{3a^2b}{3a^2b + 6ab} = \frac{3a^2b}{3ab(a+2)} = \frac{a}{a+2}
\]

Factor \( 3ab \) from numerator and denominator

Divide out (cancel) \( 3ab \)

Note: Remember that you can only divide out, or cancel, an expression if it is a factor of the numerator and denominator of a rational expression. A common mistake in this case is to assume that \( 3a^2b \) is a factor of the denominator instead of a term and “cancel” it out to obtain the incorrect simplification \( \frac{1}{6ab} \).

(b) We employ what we have learned about factoring polynomials to factor the numerator and denominator polynomials of \( \frac{x^2 - x - 6}{3x^2 + 5x - 2} \) and divide out the common factor \( x+2 \) to obtain

\[
\frac{x^2 - x - 6}{3x^2 + 5x - 2} = \frac{(x-3)(x+2)}{(3x-1)(x+2)} = \frac{x-3}{3x-1}
\]

Factor numerator and denominator polynomials

Divide out (cancel) \( x+2 \)
(c) We factor the numerator and denominator polynomials of $\frac{a^2b^3-a^2b}{a^2b+a^2}$ in two steps and divide out the common factor after each step to obtain

$$\frac{a^2b^3-a^2b}{a^2b+a^2} = \frac{a^2(b^3-b)}{a^2 \cdot (b+1)}$$

**Factor $a^2$ from numerator and denominator polynomials**

$$= \frac{b(b^3-1)}{b+1}$$

**Divide out (cancel) $a^2$; Factor $b^3-b$**

$$= \frac{b(b-1)(b+1)}{b+1}$$

**Factor $b^2-1=(b-1)(b+1)$**

$$= b(b-1)$$

**Divide out (cancel) $b+1$**

(d) We factor the numerator and denominator polynomials of $\frac{y^2-x^2}{x^2-xy}$ in two steps and divide out the common factor after the second step to obtain

$$\frac{y^2-x^2}{x^2-xy} = \frac{(y-x)(y+x)}{x(x-y)}$$

**Factor numerator and denominator polynomials**

$$= \frac{-(x-y)(y+x)}{x(x-y)}$$

**Write $y-x=-(x-y)$**

$$= \frac{-x+y}{x}$$

**Divide out (cancel) $x-y$**

Exercise Set 5.1

Find the greatest common factor (GCF) of the numerator and the denominator of each rational expression. Write the expression by factoring out the GCF in both the numerator and the denominator. Then divide out the GCF to write the rational expression in simplified form.

1. \(\frac{45a^3b^4}{9a^2b}\)
2. \(\frac{8a^{10}b^3}{6a^7b}\)
3. \(\frac{3x(x-1)^2}{5x^2(x-1)}\)
4. \(\frac{24x^3y^5}{30y^3z^2}\)
5. \(\frac{x^2(x+3)^3}{x^7(x+3)(2x-1)}\)
6. \(\frac{15(x+1)^5(x-1)}{48(x+1)^5(2x+3)}\)
Simplify the rational expression.

7. \(\frac{18x^4y^7}{24x^8y^4z}\)

9. \(\frac{4(a-b)(a+b)^2}{7(b-a)(a+b)^2}\)

11. \(\frac{x^3+3x^2}{x^2+2x^4}\)

13. \(\frac{a^2b^3+a^2b^4}{a^2b^2+a^4b^2}\)

15. \(\frac{x^3yz+xyz^3+xyz^3}{x^2y^2z^2}\)

17. \(\frac{t^3-2t^2+t}{t^2-t}\)

19. \(\frac{x^2-4}{x+2}\)

21. \(\frac{x^2+6x+8}{x^2+5x+4}\)

23. \(\frac{x^2+2x-3}{x^2+x-6}\)

25. \(\frac{4x^2-4}{12x^2+12x-24}\)

27. \(\frac{y^2-y-12}{y^2+5y+6}\)

29. \(\frac{x^2+2xy+y^2}{3x^2+2xy-y^2}\)

31. \(\frac{6x+12}{4x^2+6x-4}\)

8. \(\frac{6(x+4)^3(x-2)^2}{30(x+4)^3}\)

10. \(\frac{(x-2)(3x+5)^2}{(2-x)(3x+5)^3}\)

12. \(\frac{3x^2-15x}{12x-60}\)

14. \(\frac{x^2y}{x^2y+x^4y^2}\)

16. \(\frac{6t^4-18t^3}{4t^2-12t}\)

18. \(\frac{2a^2b^2-10a^6b^8}{2a^2b^2}\)

20. \(\frac{x^2+4x+3}{x+1}\)

22. \(\frac{(a-3)^2}{a^2-9}\)

24. \(\frac{6x+12}{x^2+5x+6}\)

26. \(\frac{4y^3+4y-8y}{2y^3+4y-6y}\)

28. \(\frac{2x^2+5x-3}{3x^2+11x+6}\)

30. \(\frac{2x^2+6xy+4y^2}{4x^2-4y^2}\)

32. \(\frac{x^7+4x^6+3x^5}{x^4+3x^3+2x^2}\)
33. \( \frac{3x^2 + 12x + 12}{9x^2 - 36} \)

34. \( \frac{t^3 + t^2}{t^2 - 1} \)

35. \( \frac{t^2 - 3t - 18}{2t^2 + 5t + 3} \)

36. \( \frac{2t^4 - t^3 - 6t^2}{2t^2 - 7t + 6} \)
5.2. Multiply and Divide Rational Expressions

**KYOTE Standards:** CR 12; CA 6

The procedures used to multiply and divide rational expressions are the same as those used to multiply and divide fractions. For example, if \( A, B, C \) and \( D \) are polynomials with \( B \neq 0 \) and \( C \neq 0 \), then

\[
\frac{A}{B} \cdot \frac{C}{D} = \frac{A \cdot C}{B \cdot D} \quad \text{and} \quad \frac{A}{B} \div \frac{C}{D} = \frac{A \cdot D}{B \cdot C}
\]

Once these operations are performed, the only remaining task is to simply the resulting rational expression as we did in Section 5.1. A few examples should illustrate this process.

**Example 1.** Perform the multiplication or division of the given rational expressions and simplify.

(a) \( \frac{x^2 + 2x - 8}{x^2 - 9} \cdot \frac{x^2 + 3x}{x^2 - 5x + 6} \)

(b) \( \frac{x^5}{5x - 10} \div \frac{x^2}{2x - 4} \)

(c) \( \frac{2a^2 - ab - b^2}{a^2 + 6ab + 9b^2} \cdot \frac{a^2 + 4ab + 3b^2}{b - a} \)

**Solution.** (a) We factor the numerators and denominators of both rational expressions in the product \( \frac{x^2 + 2x - 8}{x^2 - 9} \cdot \frac{x^2 + 3x}{x^2 - 5x + 6} \) and simplify the resulting rational expression to obtain

\[
\frac{x^2 + 2x - 8}{x^2 - 9} \cdot \frac{x^2 + 3x}{x^2 - 5x + 6} = \frac{(x-2)(x+4)}{(x-3)(x+3)} \cdot \frac{x(x+3)}{(x-2)(x-3)} = \frac{x(x+4)}{(x-3)^2}
\]

Given product of rational expressions

Factor polynomials

Divide out (cancel) \( x-2, x+3 \)

(b) We invert the second expression \( x^2 \) in the quotient \( \frac{x^5}{5x - 10} \div \frac{x^2}{2x - 4} \), multiply it by the first expression \( \frac{x^5}{5x - 10} \), and simplify to obtain

\[
\frac{x^5}{5x - 10} \div \frac{x^2}{2x - 4} = \frac{x^5}{5x - 10} \cdot \frac{2x - 4}{x^2} = \frac{x^3}{5x - 10}
\]

Given quotient of rational expressions
\[ = \frac{x^5}{5x-10} \cdot \frac{2x-4}{x^2} \qquad \text{Invert } \frac{x^2}{2x-4} \text{ and multiply} \]
\[ = \frac{x^5}{5(x-2)} \cdot \frac{2(x-2)}{x^2} \qquad \text{Factor } 2x-4, 5x-10 \]
\[ = \frac{2x^3}{5} \qquad \text{Divide out (cancel) } x-2, x^2 \]

(c) We factor the numerators and denominators of both rational expressions in the product \[ \frac{2a^2 - ab - b^2}{a^2 + 6ab + 9b^2} \cdot \frac{a^2 + 4ab + 3b^2}{b-a} \] in two steps and divide out the common factor after each step to obtain
\[ \frac{2a^2 - ab - b^2}{a^2 + 6ab + 9b^2} \cdot \frac{a^2 + 4ab + 3b^2}{b-a} \]
\[ = \frac{(2a+b)(a-b)}{(a+3b)(a+3b)} \cdot \frac{(a+b)(a+b)}{b-a} \]
\[ = \frac{(2a+b)(a-b)}{(a+3b)} \cdot \frac{a+b}{b-a} \]
\[ = \frac{-(2a+b)(b-a)}{a+3b} \cdot \frac{a+b}{b-a} \]
\[ = \frac{-2a+b}{a+3b} \]

**Exercise Set 5.2**

Perform the multiplication or division and simplify.

1. \( \frac{x^2}{4y} \cdot \frac{2x^3}{y^2} \)
2. \( -\frac{y}{8} \cdot \frac{10}{y^3} \)
3. \( \frac{a^2}{a+2} \cdot \frac{6a+12}{a^2} \)
4. \( \frac{8a-6}{5a+20} \cdot \frac{2a+8}{4a-3} \)
5. \( \frac{x^3}{y} \cdot \frac{x^5}{y^2} \)
6. \( \frac{2a+3}{a^3} \div \frac{6a+9}{a^4} \)
7. \( \frac{x-1}{(x+3)^2} \div \frac{1-x}{(x+3)^3} \)
8. \( \frac{4x-1}{3x+2} \cdot \frac{9x^2+6x}{1-4x} \)
9. \[ \frac{3x^2 \cdot x + 3}{x^2 - 9} \div 12x \]

11. \[ \frac{x^2 - x - 6 \cdot x + 1}{x^2 - 1} \div x - 3 \]

13. \[ \frac{x^2y + 3xy^2 - x^2 - 2xy - 3y^2}{x^2 - 9y^2} \div \frac{5x^3y}{x^3 - 9y^2} \]

15. \[ \frac{x^4}{x + 2} \div \frac{x^3}{x^2 + 4x + 4} \]

17. \[ \frac{2a^2 - ab - b^2 \cdot 2a^2 + ab - 3b^2}{a^2 - 2ab + b^2} \div \frac{2a^2 + 3ab + b^2}{x^2 - 4x - 5} \div \frac{x^2 + 8x + 7}{x^2 + 7x + 12} \]

10. \[ \frac{2x^2 + 7x - 4 \cdot 3 - x}{2x^2 - 3x + 1} \div \frac{x - 3}{x - 3} \]

12. \[ \frac{x^2 + 5x + 6 \cdot x^3 + x}{x^3 + 2x} \div \frac{x^2 + 4x + 3}{x^2 + 4x + 3} \]

14. \[ \frac{2x^2 + 3x + 1 \cdot x^2 + 6x + 5}{x^3 + 2x - 15} \div \frac{2x^2 - 7x + 3}{2x^2 - 7x + 3} \]

16. \[ \frac{3x^2 + 2x - 1 \cdot x^2 - 2x + 1}{x^2 - 1} \div \frac{x^2 - 2x + 1}{3x^2 - 7x + 2} \]
5.3. Add and Subtract Rational Expressions

**KYOTE Standards:** CR 12; CA 6

The procedures used to add and subtract rational expressions are the same as those used to add and subtract fractions. Two rational expressions with the same denominator are relatively easy to add and subtract. For example, if \( A, B \) and \( C \) are polynomials with \( C \neq 0 \), then

\[
\frac{A}{C} + \frac{B}{C} = \frac{A + B}{C} \quad \text{and} \quad \frac{A}{C} - \frac{B}{C} = \frac{A - B}{C}
\]

The procedure is more difficult to carry out if the denominators are different. For example, suppose \( A, B, C \) and \( D \) are polynomials with \( C \neq 0 \) and \( D \neq 0 \). To add the rational expressions \( \frac{A}{C} \) and \( \frac{B}{D} \), we must first find a common denominator, \( CD \) in this case. We then find equivalent expressions for \( \frac{A}{C} \) and \( \frac{B}{D} \) with this denominator and add to obtain

\[
\frac{A}{C} + \frac{B}{D} = \frac{AD}{CD} + \frac{BC}{CD} = \frac{AD + BC}{CD}
\]

In practice, it is important to find the least common denominator, or LCD, because otherwise the algebra becomes messy and it is difficult to reduce the rational expression that is obtained. In Section 1.2, we observed that the LCD of two or more fractions is the least common multiple, or LCM, of their denominators. We use this same approach to find the LCD of two or more rational expressions.

**Example 1.** Suppose the polynomials given are denominators of rational expressions. Find their least common denominator (LCD).

(a) \( 6x^3y^4, 8x^5y \)  
(b) \( x^2 - 9, x^2 - 2x - 15 \)  
(c) \( 5(a+1)^2, 16(a+1)^3, 10(a+1) \)

**Solution.** (a) To find the LCD of the denominators \( 6x^3y^4 \) and \( 8x^5y \), we factor \( 6 = 2 \cdot 3 \) and \( 8 = 2^3 \) into products of powers of prime numbers. We then examine these denominators in factored form:

\[
2 \cdot 3x^3y^4, \quad 2^3x^5y
\]

We view the variables \( x \) and \( y \) as prime numbers and we take the largest power of each "prime" factor \( 2, 3, x \) and \( y \) in the two expressions to form the LCD as we did in Section 1.2. The LCD of \( 6x^3y^4 \) and \( 8x^5y \) is therefore

\[
2^3 \cdot 3x^5y^4 = 24x^5y^4
\]
(b) To find the LCD of the denominators \(x^2 - 9\) and \(x^2 - 2x - 15\), we factor them to obtain \(x^2 - 9 = (x - 3)(x + 3)\) and \(x^2 - 2x - 15 = (x - 5)(x + 3)\). The LCD of \(x^2 - 9\) and \(x^2 - 2x - 15\) is therefore the product

\[(x - 3)(x + 3)(x - 5)\]

**Note.** The product \((x - 3)(x + 3)^2(x - 5)\) is a common denominator but not the least common denominator.

(c) To find the LCD of the denominators \(5(a + 1)^2\), \(16(a + 1)^3\) and \(10(a + 1)\), we factor \(16 = 2^4\) and \(10 = 2 \cdot 5\) into products of powers of primes and examine these denominators in factored form:

\[5(a + 1)^2, \quad 2^4(a + 1)^3, \quad 2 \cdot 5(a + 1)\]

We take the largest power of each “prime” factor \(2\), \(5\) and \(a + 1\), and multiply them together to obtain their LCD

\[2^4 \cdot 5(a + 1)^3 = 80(a + 1)^3\]

**Example 2.** Find the LCD of the given pair of rational expressions. Express each rational expression in the pair as an equivalent rational expression with the LCD as its denominator.

(a) \(\frac{3}{4a^2b} - \frac{5}{6ab^3}\)  
(b) \(\frac{x}{x^2 - x - 2} - \frac{x + 4}{x^2 + 3x - 10}\)

**Solution.** (a) The LCD of the denominators \(4a^2b\) and \(6ab^3\) is \(12a^2b^3\). Thus \(12a^2b^3\) is a multiple of both \(4a^2b\) and \(6ab^3\), and we can write

\[12a^2b^3 = 4a^2b \cdot 3b^2\]

\[12a^2b^3 = 6ab^3 \cdot 2a\]

We can then write \(\frac{3}{4a^2b}\) and \(\frac{5}{6ab^3}\) as equivalent rational expressions with the same denominator \(12a^2b^3\).

\[\frac{3}{4a^2b} = \frac{3 \cdot 3b^2}{4a^2b \cdot 3b^2} = \frac{9b^3}{12a^2b^3}\]

\[\frac{5}{6ab^3} = \frac{5 \cdot 2a}{6ab^3 \cdot 2a} = \frac{10a}{12a^2b^3}\]

(b) We must factor the denominator polynomials \(x^2 - x - 2 = (x - 2)(x + 1)\) and \(x^2 + 3x - 10 = (x - 2)(x + 5)\) to find their LCD. Their LCD is therefore \((x - 2)(x + 1)(x + 5)\).
We can then write \( \frac{x}{x^2 - x - 2} \) and \( \frac{x + 4}{x^2 + 3x - 10} \) as equivalent rational expressions with the same denominator \((x - 2)(x + 1)(x + 5)\).

\[
\frac{x}{x^2 - x - 2} = \frac{x}{(x - 2)(x + 1)} = \frac{x(x + 5)}{(x - 2)(x + 1)(x + 5)}
\]

\[
\frac{x + 4}{x^2 + 3x - 10} = \frac{x + 4}{(x - 2)(x + 5)} = \frac{(x + 4)(x + 1)}{(x - 2)(x + 5)(x + 1)}
\]

**Example 3.** Perform the addition or subtraction and simplify. Identify the *LCD* in each case.

(a) \( \frac{5x}{6} + \frac{3x}{8} \)  
(b) \( \frac{1}{2a} + \frac{1}{3a^2} \)  
(c) \( \frac{2}{x - 2} - \frac{3}{x + 1} \)

**Solution.** (a) The *LCD* of the denominators 6 and 8 is 24. Thus 24 is a multiple of both 6 and 8, and we can write 24 = 6 \cdot 4 and 24 = 8 \cdot 3. We write both \( \frac{5x}{6} \) and \( \frac{3x}{8} \) as equivalent expressions with the same denominator of 24 and add to obtain

\[
\frac{5x}{6} + \frac{3x}{8} = \frac{5x \cdot 4}{6 \cdot 4} + \frac{3x \cdot 3}{8 \cdot 3}
\]

Write each term as an equivalent expression with *LCD* 24

\[
= \frac{20x}{24} + \frac{9x}{24}
\]

Simplify

\[
= \frac{29x}{24}
\]

Add

(b) The *LCD* of the denominators 2a and 3a\(^2\) is 6a\(^2\). Thus 6a\(^2\) is a multiple of both 2a and 3a\(^2\), and we can write 6a\(^2\) = 2a \cdot 3a and 6a\(^2\) = 3a\(^2\) \cdot 2. We write both \( \frac{1}{2a} \) and \( \frac{1}{3a^2} \) as equivalent expressions with the same denominator of 6a\(^2\) and add to obtain

\[
\frac{1}{2a} + \frac{1}{3a^2} = \frac{1 \cdot 3a}{2a \cdot 3a} + \frac{1 \cdot 2}{3a^2 \cdot 2}
\]

Write each term as an equivalent expression with *LCD* 6a\(^2\)

\[
= \frac{3a}{6a^2} + \frac{2}{6a^2}
\]

Simplify

\[
= \frac{3a + 2}{6a^2}
\]

Add
(c) The LCD of the denominators \( x - 2 \) and \( x + 1 \) is \((x - 2)(x + 1)\). We write both \( \frac{2}{x - 2} \) and \( \frac{3}{x + 1} \) as equivalent expressions with the same denominator of 
\((x - 2)(x + 1)\), subtract and simplify to obtain

\[
\frac{2}{x - 2} - \frac{3}{x + 1} = \frac{2(x + 1)}{(x - 2)(x + 1)} - \frac{3(x - 2)}{(x + 1)(x - 2)}
\]

Write each term as an equivalent expression with LCD \((x - 2)(x + 1)\)

\[
= \frac{2(x + 1) - 3(x - 2)}{(x - 2)(x + 1)}
\]

Subtract

\[
= \frac{2x + 2 - 3x + 6}{(x - 2)(x + 1)}
\]

Expand numerator expressions

\[
= \frac{-x + 8}{(x - 2)(x + 1)}
\]

Collect like terms

Example 4. Perform the addition or subtraction and simplify. Identify the LCD in each case.

(a) \( \frac{2y}{y+1} + \frac{3}{y+1} \)

(b) \( \frac{1}{x^2 + 4x + 4} - \frac{x+1}{x^2 - 4} \)

(c) \( \frac{3}{t} - \frac{2}{t+2} + \frac{4}{t^2 + 2t} \)

Solution. (a) The term \( \frac{2y}{y+1} \) is a rational expression with denominator 1. The LCD of the denominators 1 and \( y+1 \) is \( y+1 \). We write both \( \frac{2y}{y+1} \) and \( \frac{3}{y+1} \) as equivalent expressions with the same denominator of \( y+1 \) and add to obtain

\[
\frac{2y}{y+1} + \frac{3}{y+1} = \frac{2y(y+1)}{y+1} + \frac{3}{y+1}
\]

Write each term as an equivalent expression with LCD \( y+1 \)

\[
= \frac{2y^2 + 2y + 3}{y+1}
\]

Expand \( 2y(y+1) \) and add

(b) We must factor the denominator polynomials \( x^2 + 4x + 4 = (x+2)^2 \) and \( x^2 - 4 = (x - 2)(x + 2) \) to find their LCD \((x+2)^2(x-2)\). We write both \( \frac{1}{x^2 + 4x + 4} \) and \( \frac{x+1}{x^2 - 4} \) as equivalent expressions with the same denominator of \((x+2)^2(x-2)\), subtract and simplify to obtain
\[
\frac{1}{x^2+4x+4} - \frac{x+1}{x^2-4} = \frac{1}{(x+2)^2} - \frac{x+1}{(x-2)(x+2)}
\]

\[
= \frac{x-2}{(x+2)^2(x-2)} - \frac{(x+1)(x+2)}{(x-2)(x+2)(x+2)}
\]

\[
= \frac{x-2-(x+1)(x+2)}{(x-2)(x+2)^2}
\]

\[
= \frac{x-2-(x^2+3x+2)}{(x-2)(x+2)^2}
\]

\[
= \frac{-x^2-2x-4}{(x-2)(x+2)^2}
\]

**Factor denominator polynomials**

Write each term as an equivalent expression with LCD \((x+2)^2(x-2)\)

**Subtract**

Multiply \((x+1)(x+2)=x^2+3x+2\)

**Collect like terms**

\[(c)\] We factor \(t^2+2t=t(t+2)\) and we see that the LCD of the denominators \(t\), \(t+2\) and \(t(t+2)\) is \(t(t+2)\). We write \(\frac{3}{t}\), \(\frac{2}{t+2}\) and \(\frac{4}{t(t+2)}\) as equivalent expressions with the same denominator of \(t(t+2)\), subtract and add the numerators, and simplify to obtain

\[
\frac{3}{t} - \frac{2}{t+2} + \frac{4}{t^2+2t} = \frac{3}{t} - \frac{2}{t+2} + \frac{4}{t(t+2)}
\]

\[
= \frac{3(t+2)}{t(t+2)} - \frac{2t}{(t+2)t} + \frac{4}{t(t+2)}
\]

\[
= \frac{3t+6-2t+4}{t(t+2)}
\]

\[
= \frac{t+10}{t(t+2)}
\]

**Factor** \(t^2+2t=t(t+2)\)

Write each term as an equivalent expression with LCD \(t(t+2)\)

Multiply \(3(t+2)=3t+6\); subtract and add

Collect like terms

**Exercise Set 5.3**

Suppose the expressions given are denominators of rational expressions. Find their least common denominator (LCD).

1. \(x^4y^5, x^2y^7z\)

2. \(2ab^5, 3a^6b^3\)

3. \(12(x-1), 9(x-1)^3\)

4. \((x+2)^3(x+3), (x+3)^2(x+4), (x+4)^5\)

5. \(x^2+5x+6, (x+2)^2\)

6. \(15x^2(y+1), 21x(y+1)^3\)
7. \(x^2 - 25, \ x^2 + 8x + 15\)  

8. \(t(t^2 - 1), \ t^3(t + 1), \ t^2(t - 1)\)  

Write each pair of rational expressions as equivalent rational expressions with their LCD as the denominator for both.

9. \(\frac{7}{2x^2}, \ \frac{5}{3x^2}\)  

10. \(\frac{1}{4ab^2}, \ \frac{1}{a^3b}\)  

11. \(\frac{1}{12x^2y^3}, \ \frac{1}{18x^3y}\)  

12. \(\frac{x+1}{4xy}, \ \frac{5y}{6x^2}\)  

13. \(\frac{3}{x(x-1)}; \ \frac{7}{(x-1)^2}\)  

14. \(\frac{x}{4x+4}; \ \frac{1}{x^2-1}\)  

15. \(\frac{x}{x^2+4x+3}, \ \frac{x+5}{x^2+3x+2}\)  

16. \(\frac{x+3}{x^2-x-2}, \ \frac{6x}{x^2-4x+4}\)  

Perform the addition or subtraction and simplify. Identify the LCD in each case.

17. \(\frac{y}{7x^2} - \frac{4}{7x^2}\)  

18. \(\frac{4}{(x+1)^2} + \frac{9}{(x+1)^2}\)  

19. \(\frac{x}{4} + \frac{2x}{3}\)  

20. \(\frac{1}{2a} + \frac{4}{3a}\)  

21. \(\frac{1}{s} + \frac{1}{t}\)  

22. \(\frac{2}{3x} - \frac{1}{6y}\)  

23. \(\frac{3}{c^3} - \frac{4}{c^4}\)  

24. \(\frac{5}{6a^2b} + \frac{2}{9ab^2}\)  

25. \(\frac{3}{x} + \frac{5}{x+2}\)  

26. \(\frac{1}{x-1} - \frac{1}{x+1}\)  

27. \(5 + \frac{4}{x-2}\)  

28. \(\frac{3x}{x^2-4} - \frac{1}{x-2}\)  

29. \(\frac{1}{2r} + \frac{1}{3s} + \frac{1}{4t}\)  

30. \(\frac{2x-1}{3} - \frac{x-4}{5}\)
31. \( x - \frac{x}{x+7} \)

32. \( \frac{3 + \frac{2}{x}}{x-1} - \frac{5}{x^2-x} \)

33. \( \frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3} \)

34. \( \frac{x}{x^3-x^2} + \frac{1}{x^3} \)

35. \( \frac{3}{x-5} + \frac{4}{5-x} \)

36. \( \frac{x}{x^2 + x - 6} + \frac{x+1}{x^2 + 7x + 12} \)

37. \( \frac{2}{x^2 + 2x - 15} - \frac{1}{x^2 - 5x + 6} \)

38. \( \frac{7x - 1}{x^2 - 9x + 20} - \frac{1}{x-5} \)

39. \( \frac{1}{x+1} - \frac{2}{(x+1)^2} + \frac{3}{x^2 - 1} \)

40. \( \frac{t+1}{2t^2 + 5t - 3} + \frac{t+3}{2t^2 - 3t + 1} \)